Topic 5 Outline

1. Related Rates
   - Solving Related Rates Problems

Topic 5 Learning Objectives

1. recall geometric formulas for areas, perimeters, surface areas, volumes, right angle triangles, similar triangles, etc
2. set up related rates equations based on word problems
3. differentiate equations implicitely with respect to time
4. solve related rates problems

Related Rates

In this section we work on (word) problems where we need to compute the rate of change of one quantity in terms of the rate of change of other quantities. The process is to find an equation that relates the two (or more) quantities, and then to use the chain rule to differentiate both sides (with respect to time) and solve for the desired rate of change.

Check out this link for a video on related rates!
https://www.educreations.com/lesson/embed/9752155/?ref=app

Strategy

1. Read the problem slowly and carefully.
2. Draw a diagram.
3. Assign symbols to all quantities that are a function of time.
4. Identify "what do we know" to write an equation that relates the various quantities of the problem.
5. Identify "what do we need".
6. Differentiate both sides of your equation with respect to the variable t.
7. Plug in all of the info you were given to solve for the missing rate!
8. Be sure to finish with a sentance, including units.

Problem 1

Air is being pumped into a spherical balloon so that its volume increases at a rate of $100cm^3/s$. How fast is the radius of the balloon increasing when the diameter is $50cm$?

Problem 2

A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6ft away from the wall?

Problem 3

A water tank has the shape of an inverted circular come with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2m^3/\text{min}$, find the rate at which the water lebel is rising when the water is 3m deep.

Problem 4

A particle moves along the curve $y = -x^2 + 7$. As it passes through the poit $(1, 6)$ its x-coordinate increases at a rate of 2cm/s. How fast is the distance from the particle to the origin changing tat this instant?

Problem 5

A street light is mounted at the top of a 15ft pole. A man 6ft tall walks away from the pole at a rate of 5ft per second. How fast is the tip of his shadow moving when he is 40ft from the pole?

Related Rates

The circumstances look grim. But Ethan felt assured of his survival thanks to the many years of algebra word problems which dealt with this exact scenario.

Five in Five!

A balloon in the shape of a perfect cube is deflating as time goes on, at a rate of $48cm^3/s$.

1. What is the formula for the volume of the cube?
2. Differentiate the formula for the volume of a cube.
3. At the moment when the length of its side is 2cm, what it the rate at which the length of a side is changing?
4. What is the formula for the surface area of the cube?
5. At the moment mentioned above, what is the rate at which the surface area of the balloon is changing?

Flex the Mental Muscle!

A man walks along a straight path at a speed of 4ft/s. A searchlight is located on the ground 20ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15ft from the point on the path closest to the searchlight?